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FIXED POINT RESULTS ON PERTURBED B-METRIC SPACE

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In this paper, we introduce the notion of perturbed b-metric spaces and consider some basic topological notions on this new abstract structure. In addition, we shall discuss the existing and uniqueness of a fixed point in the setting of perturbed b-metric spaces.

1. INTRODUCTION AND PRELIMINARIES

One of the most useful concepts in calculating physical phenomena is undoubtedly the concept of metric. We are quite familiar with the concept of metric, which was introduced by Frechet more than a century ago. The reason for this is that the concept of metric is actually a generalization of the concept of distance, which was axiomatically established by Euclid, who lived in 325 BC - 265 BC.

In fact, the concept of metric was seen as sufficient in understanding, evaluating, and interpreting the physical problems and phenomena of the 19th and even 20th centuries. However, when the speed of scientific developments and advances in the last century is taken into account, in-depth calculations and studies begin to be encountered in which the concept of metric, which we can consider a kind of "perfect," is inadequate. This led to the need to re-evaluate the concept of metric and generalize, improve, and extend this concept. Consequently, researchers have considered suggesting new concepts to understand the physical phonema better.

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As a result of this trend, in the literature, several distinct extensions of the metric concept have been released, such as b-metric, quasi-metric, G-metric, fuzzy metric, and so on; see e.g., [**7, 1**]. Among all, the recently introduced notion of perturbed metric [**6**] should be comprised, see also [**2, 3, 8**]. Indeed, the idea is very natural and reasonable. In real life, not all problems can be transferred into a perfect framework to be solved.

Perturbation theory in applied mathematics brings methods for approximating a solution to a problem by starting from the exact solution of a simpler, related problem. In these cases, we try to move this existing perturbed state into one of the proper states we know. This is basically like the approximation method. We look at the problem carefully to separate the problem into solvable part and the perturbed part. Roughly speaking, this is the main idea of the perturbed metric.

In this paper, we generalize and extend the notion of perturbed metric spaces under the name of perturbed b-metric spaces. Furthermore, we shall discuss the existence and uniqueness of fixed points of certain operators in the context of perturbed b-metric spaces.

Throughout this paper, let X represent an arbitrary non-empty set, and $\mathbb{N}, \mathbb{R}$ denote the set of positive integers and real numbers, respectively. We reserve the symbol $\mathbb{N}_0$ for nonnegative integers, more precisely $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

The following definition introduces the recently proposed concept of perturbed metric spaces by Jleli and Samet [**6**]:

Definition 1. *[**6**] Let $\Delta, Q : X \times X \to [0, \infty)$ be two given mappings. We say that Δ is a perturbed metric on X with respect to Q, if*

$$\begin{gathered} \Delta - Q : X \times X \to \mathbb{R}, \\ (x, y) \longrightarrow \Delta(x, y) - Q(x, y) \end{gathered}$$

is a metric on X, that is, for all $x, y, z \in X$,

(i) $(\Delta - Q)(x, y) \geq 0$;

(ii) $(\Delta - Q)(x, y) = 0$ *if and only if* $x = y$;

(iii) $(\Delta - Q)(x, y) = (\Delta - Q)(y, x)$;

(iv) $(\Delta - Q)(x, y) \leq (\Delta - Q)(x, z) + (\Delta - Q)(z, y)$.

The mapping Q is called a perturbed mapping, where $\delta_Q = \Delta - Q$ is a standard metric. In addition, the triple (X, Δ, Q) is called a perturbed metric space.

A perturbed metric on X does not need to be a metric on X. For this purpose, let us consider a standard metric space (X, ρ) . It is enough to consider $\Delta : X \times X \to [0, \infty)$ as $\Delta(x, y) = \rho(x, y) + (x + y)^2$ for all $x, y \in X$. It is obvious that for the perturbed mapping, $Q : X \times X \to [0, \infty)$ with $Q(x, y) = (x + y)^2$, we have a standard metric $\delta_Q(x, y) = (\Delta - Q)(x, y)$ for all $x, y \in X$. On the other hand, Δ does not form a metric, since $\Delta(2, 2) = 16 \neq 0$.

Definition 2. *Let $D, P : X \times X \to [0, \infty)$ be two given mappings and $s \geq 1$. We say that D is a perturbed b-metric on X with respect to P, if*

$$D - P : X \times X \to \mathbb{R},$$
$$(x, y) \longrightarrow D(x, y) - P(x, y)$$

is a b-metric on X, that is, for all $x, y, z \in X$,

(i) $(D - P)(x, y) \geq 0$;

(ii) $(D - P)(x, y) = 0$ *if and only if* $x = y$;

(iii) $(D - P)(x, y) = (D - P)(y, x)$;

(iv) $(D - P)(x, y) \leq s[(D - P)(x, z) + (D - P)(z, y)]$.

The mapping P is called a perturbed mapping, $d_P = D - P$ a standard b-metric, and (X, D, P) a perturbed b-metric space.

Remark 1. *A perturbed b-metric on X is not necessarily a b-metric or a standard metric on X. To illustrate this claim, it is sufficient to define $D : X \times X \to [0, \infty)$ as $D(x - y) = |x - y|^2 + (x + y)^2$ for all $x, y \in X$. It is clear that for the perturbed mapping, $P : X \times X \to [0, \infty)$ with $P(x, y) = (x + y)^2$, we have a standard b-metric $d(x, y) = |x - y|^2 = (D - P)(x, y)$ for all $x, y \in X$. Note also that $d(x, y) = |x - y|^2$ is not a standard metric, either. Furthermore, D does not form neither a b-metric, nor a metric, since $D(1, 1) = 4 \neq 0$.*

Remark 2. *It is straightforward to conclude that each perturbed b-metric space is a perturbed metric. The converse is false due to the example that is considered in the remark above.*

In what follows, we shall state the basic topological properties of the perturbed b-metric spaces:

Definition 3. *Let the triple (X, D, P) be a perturbed b-metric space and $\{\xi_n\}$ be a sequence in X, and $T : X \to X$.*

(i) We say that $\{\xi_n\}$ is a perturbed convergent sequence in (X, D, P), if $\{\xi_n\}$ is a convergent sequence in the b-metric space (X, d), where d is the standard b-metric (i.e., $d = D - P$).

(ii) We say that $\{\xi_n\}$ is a perturbed Cauchy sequence in (X, D, P), if $\{\xi_n\}$ is a Cauchy sequence in the b-metric space (X, d).

(iii) We say that (X, D, P) is a complete perturbed b-metric space, if (X, d) is a complete b-metric space or, equivalently, if every perturbed Cauchy sequence in (X, D, P) is a perturbed convergent sequence in (X, D, P).

(iv) We say that T is a perturbed continuous mmapping if it T is continuous with respect to the exact b-metric d.

Lemma 1. *Let $D, P, Q : X \times X \to [0, \infty)$ be three given mappings and $\tau > 0$.*

(i) If (X, D, P) and (X, D, Q) are perturbed b-metric spaces, then $\left(X, D, \frac{P+Q}{2}\right)$ is a perturbed metric space.

(ii) If (X, D, P) is a perturbed b-metric space, then $(X, \tau D, \tau P)$ is a perturbed metric space.

The proof of Lemma 1 is straightforward, so we skip it.

In the following theorem, Jleli and Samet [**6**] obtained an analog of Banach's fixed point theorem in the setting of perturbed metric spaces.

Theorem 2. *Let (X, D, P) be a complete perturbed metric space and $T : X \to X$ be a given mapping. Assume that the following conditions hold:*

(i) T is a perturbed continuous mapping;

(ii) There exists $\lambda \in (0, 1)$ such that

$$D(Tu, Tv) \leq \lambda D(u, v),$$

for all $u, v \in X$.

Then T admits one and only one fixed point.

This is the first fixed-point result in the context of perturbed metric spaces.

In what follows, we collect the following interesting auxiliary functions and properties of these interesting functions, for more details, see e.g. [**5, 4, 9**].

Definition 4. *A mapping $\varphi : [0, \infty) \to [0, \infty)$ is called a comparison function if it is increasing and $\varphi^m(t) \to 0$, as $m \to \infty$, for any $t \in (0, \infty)$, where $\varphi^m(t)$ is m^{th} iteration of $\varphi(t)$.*

Lemma 3. *([**5, 9**]) If $\varphi : [0, \infty) \to [0, \infty)$ is a comparison function, then:*

(1) each iterate φ^k of φ, $k \geq 1$, is also a comparison function,

(2) φ is continuous at 0,

(3) $\varphi(t) < t$, for any $t > 0$.

Definition 5. *([**5, 9**]) An increasing function $\varphi : [0, \infty) \to [0, \infty)$ is said to be a (c)-comparison function if there exist $k_0 \in \mathbb{N}$, $a \in (0, 1)$ and a convergent series of non-negative terms $\sum_{k=1}^{\infty} v_k$ such that $\varphi^{k+1}(t) \leq a\varphi^k(t) + v_k$, for $k \geq k_0$ and any $t \in [0, \infty)$.*

Definition 6. *([**4**]) Let $s \geq 1$ be a real number. A monotone increasing mapping $\varphi : [0, \infty) \to [0, \infty)$ is called a (b)-comparison function if there exist $k_0 \in \mathbb{N}$, $a \in (0, 1)$ and a convergent series of nonnegative terms $\sum_{k=1}^{\infty} v_k$ such that*

$$s^{k+1}\varphi^{k+1}(t) \leq as^k\varphi^k(t) + v_k,$$

for $k \geq k_0$ and any $t \in [0, \infty)$.

It is obvious that the concept of the (b)-comparison function reduces to that of (c)-comparison function when $s = 1$.

The following lemma is very crucial in the proof of our results.

Lemma 4. ([9]) *If $\varphi : [0,\infty) \to [0,\infty)$ is a (b)-comparison function, then we have the following conclusions:*

(1) the series $\sum_{k=0}^{\infty} s^k \varphi^k(t)$ converges for any $t \in [0,\infty)$;

(2) the function $S_b : [0,\infty) \to [0,\infty)$ defined by $S_b(t) = \sum_{k=0}^{\infty} s^k \varphi^k(t)$, $t \in [0,\infty)$, is increasing and continuous at 0.

We note that any (b)-comparison function is a comparison function due to the above lemma.

2. MAIN RESULTS

Definition 7. *Let (X, D, P) be a perturbed b-metric space, $T : X \to X$ a mapping and $\psi : [0,\infty) \to [0,\infty)$ a (b)-comparison function. Then T is called a perturbed ψ-contraction if the following condition is satisfied:*

$$D(Tx, Ty) \leq \psi(D(x, y)), \tag{2.1}$$

for all $x, y \in X$.

Proposition 5. *Let (X, D, P) be a perturbed b-metric space, and $T : X \to X$ be a perturbed ψ-contraction. Then,*

(i) the fixed point of T in X is unique provided it exists.

(ii) T is asymptotically regular at every $x \in X$.

Proof. We shall provide the proofs, case by case. Proof of (i): We will employ the approach of *Reductio ad Absurdum* to demonstrate the uniqueness of the fixed point of T. Suppose, on the contrary, there are two distinct fixed points $x^*, x^\dagger \in X$ of the mapping T. Thus, $Tx^* = x^* \neq x^\dagger = Tx^\dagger$ and necessarily, we have $D(Tx^*, Tx^\dagger) > 0$.

Since T is a perturbed ψ-contraction, together with the Lemma 3, we derive that

$$0 < D(x^*, x^\dagger) = D(Tx^*, Tx^\dagger) \leq \psi(D(x^*, x^\dagger)) < D(x^*, x^\dagger)),$$

a contradiction. Hence, our assertion is false, and T admits a unique fixed point.

Proof (ii) Let $x \in X$ be arbitrary. Case 1. Suppose that for some iteration $i \in \mathbb{N}$ of T, we have $T^i x = T^{i-1} x$, then the proof is completed. More precisely, the inequality above can be written as $T(T^{i-1}x) = T^i x = T^{i-1}x$, which indicates $T^{i-1}x$ is a fixed point of T.

Due to this observation, we easily state the following line:

$$T^{i+n}x = T^{n+1}(T^{i-1}x) = T(T(T\cdots T(T^{i-1}x))) = T^{i-1}x,$$

for any $n \in \mathbb{N}$. Thus, for any sufficiently big natural number m, we can consider that there is a natural number $n \in \mathbb{N}$ such that $m = n + i$. Consequently, we have

$$\begin{aligned} D(T^m x, T^{m-1}x) &= D(T^{n+1+i-1}x, T^{n+i-1}x)) \\ &= D(T^{n+1}(T^{i-1}x), T^{n+1}(T^{i-1}x))) = D(T^{i-1}x, T^{i-1}x) = 0. \end{aligned}$$

In other words,

$$\lim_{m\to\infty} D(T^m x, T^{m-1}x) = \lim_{m\to\infty} D(T^{i-1}x, T^{i-1}x) = 0.$$

Case 2. Suppose that Case 1 fails. In other words, for any $i \in \mathbb{N}$, we have $T^i x \neq T^{i+1}x$ for any $x \in X$, that is, $D(T^{i+1}x, T^i x) > 0$ for all $i \in \mathbb{N}$. This is a simple but important observation to be able to use Lemma 3. Employing the fact that T is a perturbed ψ-contraction, for any $1 \leq m \in \mathbb{N}$ we find

$$D(T^m x, T^{m-1}x) \leq \psi(D(T^{m-1}x, T^{m-2}x)). \tag{2.2}$$

Recursively, we obtain from the inequality above that

$$D(T^m x, T^{m-1}x) \leq \psi^m(D(T^1 x, T^0 x)), \tag{2.3}$$

where $T^0 = T$. By Lemma 3, (1), we find that

$$\lim_{m\to\infty} D(T^m x, T^{m+1}x) \leq \lim_{m\to\infty} \psi^m(D(T^1 x, T^0 x)) = 0.$$

In any case, we observe that T is asymptotically regular at every $x \in X$. □

Theorem 6. *Let (X, D, P) be a complete perturbed b-metric space, and $T : X \to X$ be a perturbed ψ-contraction. If T is a perturbed continuous mapping, then T possesses a unique fixed point u in X and for every $x_0 \in X$, the Picard sequence $\{x_m\}$, where $x_m = Tx_{m-1}$ for all $n \in \mathbb{N}_0$ converges to the fixed point of T.*

Proof. Take any $x \in X$ as an initial term and rename it as $x_0 := x$. A recursive sequence will be constructed by setting the general term to $x_m := Tx_{m-1} = T^m x_0$ be used for any $n \in \mathbb{N}$. On account of Proposition 5, we deduce that the mapping T is asymptotically regular at all $x \in X$. Moreover, from the proof of Proposition 5, we know that

$$\begin{aligned} D(x_{m+1}, x_m) &= D(T^m x, T^{m-1}x) \leq \psi(D(T^{m-1}x, T^{m-2}x)) \\ &= \psi(D(x_m, x_{m-1})) < D(x_m, x_{m-1}), \end{aligned} \tag{2.4}$$

for any $m \in \mathbb{N}$. In other words, the constructed sequence $\{D(x_m, x_{m-1})\}$ is monotonically nondecreasing. Furthermore, the recursion of the inequality (2.4) implies that

$$\begin{aligned} D(x_{m+1}, x_m) &= D(T^m x, T^{m-1}x) \leq \psi(D(T^{m-1}x, T^{m-2}x)) \\ &\leq \psi^m(D(T^1 x, T^0 x)) = \psi^m(D(x_1, x_0)) \end{aligned} \tag{2.5}$$

So, we infer, from the inequality (2.4), that the sequence of nonnegative reals $\{D(T^m x, T^{m-1}x)\}$ decreases monotonically. Consequently, one can deduce that this sequence is convergent. Suppose that κ is the limit of the sequence; that is,

$$\lim_{m\to\infty} D(T^m x, T^{m+1}x) = \kappa \geq 0.$$

We claim that $\kappa = 0$. Suppose, on the contrary, that $\kappa > 0$. Letting lim sup of both sides of (2.4) as $m \to \infty$, we find

$$0 \leq \kappa \leq \psi(\kappa) < \kappa,$$

a contradiction. Thus, we find

$$\lim_{m\to\infty} D(T^m x, T^{m-1}x) = \kappa = 0.$$

Next, we shall demonstrate that the constructed sequence $\{T^m x_0\} = \{\ x_m\}$ is Cauchy. For this purpose, we need estimations on the regular and perturbed parts of the perturbed b-metric.

Due to the nature of perturbed b-metric D, there is a standard b-metric part $d(x, y)$ and perturbed part $P(x, y)$, that is, $d(x, y) = D(x, y) - P(x, y)$ or equivalently, we have

$$D(x, y) = d(x, y) + P(x, y),$$

for all $(x, y) \in X^2$. On account of (2.8), we have

$$\begin{aligned} d(x_{m+1}, x_m) + P(x_{m+1}, x_m) &= D(x_{m+1}, x_m) = D(T^m x, T^{m-1}x) \\ &\leq \psi(D(T^{m-1}x, T^{m-2}x)) \leq \psi^m(D(x_1, x_0)) \end{aligned} \tag{2.6}$$

Set $\Theta := D(x_1, x_0)$. Keeping in mind that the function ψ is increasing, together with the observation above, one can naturally conclude the following estimations:

$$d(x_{m+1}, x_m) \leq d(x_{m+1}, x_m) + P(x_{m+1}, x_m) \leq \psi^m(\Theta), \tag{2.7}$$

and

$$P(x_{m+1}, x_m) \leq d(x_{m+1}, x_m) + P(x_{m+1}, x_m) \leq \psi^m(\Theta). \tag{2.8}$$

$$\begin{aligned} d(x_m, x_{m+p}) &\leq sd(x_m, x_{m+1}) + s^2 d(x_{m+1}, x_{m+2}) + ... + s^{p-2} d(x_{m+p-3}, x_{m+p-2}) \\ &\quad + s^{p-1} d(x_{m+p-2}, x_{m+p-1}) + s^p d(x_{m+p-1}, x_{m+p}) \\ &\leq s\psi^m(\Theta) + s^2\psi^{m+1}(\Theta) + ... + s^{p-2}\psi^{m+p-3}(\Theta) \\ &\quad + s^{p-1}\psi^{m+p-2}(\Theta) + s^{p-1}\psi^{m+p-1}(\Theta) \\ &= \tfrac{1}{s^{m-1}}[s^m\psi^m(\Theta) + s^{m+1}\psi^{m+1}(\Theta) + ... + s^{m+p-2}\psi^{m+p-2}(\Theta) \\ &\quad + s^{m+p-1}\psi^{m+p-1}(\Theta)]. \end{aligned}$$

Denoting $\tau_m = \sum_{k=0}^{m} s^k\psi^k(\Theta)$, $m \geq 1$ we obtain:

$$d(x_m, x_{m+p}) \leq \frac{1}{s^{m-1}}[\tau_{m+p-1} - \tau_{m-1}],\ n \geq 1,\ p \geq 1. \tag{2.9}$$

Due to Lemma 4, we conclude that the series $\sum_{k=0}^{m} s^k\psi^k(d(x_0, x_1))$ is convergent. Thus, there exists $\tau = \lim_{n\to\infty} \tau_m \in [0, \infty)$. Regarding $s \geq 1$ and by (2.9), we obtain that $\{x_m\}_{n\geq 0}$ is a Cauchy sequence in the b-metric space (X, d). By definition, it is a perturbed Cauchy sequence in (X, D, P).

Regarding the fact that (X, D, P) is a complete perturbed b-metric space, we conclude that there exists $x^* \in X$ such that

$$\lim_{m\to\infty} d(x_m, x^*) = 0.$$

We can easily find

$$\lim_{m\to\infty} d(Tx_m, Tx^*) = 0 \Leftrightarrow \lim_{m\to\infty} d(x_{m+1}, Tx^*) = 0,$$

since T is a perturbed continuous mapping.

Recalling the fact that $d = D - P$ is a standard b-metric on X, we find

$$Tx^* = x^*,$$

that is, x^* is a fixed point of T, by the uniqueness of the limit in (X, d).

Note that the uniqueness follows from Proposition 5. □

Example 1. *Consider $X = [1, \infty)$ and the mappings $P, D : X \times X \to [0, \infty)$ that are defined by*

$$D(x, y) = |x - y|^2 + \frac{x^2y^2}{1 + |x - y|},$$

and

$$P(x,y) = \frac{x^2y^2}{1+|x-y|}$$

and $d(x,y) = |x-y|^2$ *is a standard b-metric space with a constant* $s = 2$.

We consider the mapping $T : X \to X$ *defined by* $T(x) = \dfrac{\sqrt{x}}{2}$.

Clearly, the triple (X, D, P) *forms a perturbed b-metric space.*

It is worthful to underline that D *is neither a metric nor a b-metric on* X. *Indeed,* $D(1,1) = 1 \neq 0$.

By letting $\psi(t) = \frac{t}{2}$ *one can easily demonstrate that* T *fulfills all requirements of Theorem 6. It is evident that* $x = 1$ *is the required unique fixed point of* T.

3. CONSEQUENCES

In this section, we shall mention that Theorem 6 has several consequences, and we shall indicate only some basic ones.

Corollary 7. *Let* (X, D, P) *be a complete perturbed b -metric space. Suppose that there are* $k \in [0,1)$*, and a self-mapping* $T : X \to X$ *fulfills the inequality*

$$D(Tx,Ty) \leq kD(x,y) \text{ for all } x, y \in X. \tag{3.10}$$

If T *is perturbed continuous mapping, then* T *possesses a unique fixed point* u *in* X *and for every* $x_0 \in X$ *recursive sequence* $\{x_m\}$*, where* $x_m = Tx_{m-1}$ *for all* $m \in \mathbb{N}_0$ *converges to the fixed point of* T.

Proof. It is sufficient to take $\psi(t)) = kt$, for all $t \in [0,\infty)$ and mentioned $k \in [0,1)$ in Definition 7 . The rest is a mimic of the proof of Theorem 6. □

Corollary 8. *Let* (X, D, P) *be a complete perturbed metric space. Suppose that there are* (c)*-comparison function* ψ*, and a self-mapping* $T : X \to X$ *fulfills the inequality*

$$D(Tx,Ty) \leq \psi(D(x,y)) \text{ for all } x, y \in X. \tag{3.11}$$

If T *is a perturbed continuous mapping, then* T *possesses a unique fixed point* u *in* X *and for every* $x_0 \in X$ *recursive sequence* $\{x_m\}$*, where* $x_m = Tx_{m-1}$ *for all* $m \in \mathbb{N}_0$ *converges to the fixed point of* T.

Proof. The proof follows from verbatim of the proof of Theorem 6. Indeed, we take $s = 1$ everywhere in the statements and in the proof of Theorem 6. □

Corollary 9. *Let (X, D, P) be a complete perturbed metric space. Suppose that there are $k \in [0, 1)$, and a self-mapping $T : X \to X$ fulfills the inequality*

$$D(Tx, Ty) \leq kD(x, y) \text{ for all } x, y \in X. \tag{3.12}$$

If T is perturbed continuous mapping, then T possesses a unique fixed point u in X and for every $x_0 \in X$ recursive sequence $\{x_m\}$, where $x_m = Tx_{m-1}$ for all $m \in \mathbb{N}_0$ converges to the fixed point of T.

Proof. It is sufficient to take $\psi(t)) = kt$ for all $t \in [0, \infty)$ and mentioned $k \in [0, 1)$ in Corollary 8. The rest is a verbatim of the proof of Corollary 8. □

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