

CORRIGENDUM TO “COMMON FIXED POINTS FOR ENRICHED g -CONTRACTION MAPPINGS”

Manahell Alsosui* , Naseer Shahzad  and Amer Hassan Albargi 

This corrigendum repairs the auxiliary construction used in the proof of Theorem 7 and corrects some related points in Theorems 5, 6, 8, and 9 and Corollary 2.

In the proof of Theorem 7 in our article *Common Fixed Points for Enriched g -Contraction Mappings*, *Applied Analysis and Discrete Mathematics* **19** (2025), 577–594, the auxiliary mappings were constructed using a sequence $\{k_n\} \subset (0, 1)$ satisfying $k_n \rightarrow 1$, together with a contractive restriction depending on b . For $b > 0$, these requirements are not compatible. The statement of Theorem 7 remains unchanged; only the auxiliary construction in its proof is replaced by the following argument.

Put

$$\lambda = \frac{1}{b+1}, \quad f_{g,\lambda} = (1-\lambda)g + \lambda f.$$

The enriched g -nonexpansive condition (8) in the original article gives, for all $u, v \in \mathcal{M}$,

$$\begin{aligned} \|f_{g,\lambda}u - f_{g,\lambda}v\| &= \lambda \|b(gu - gv) + fu - fv\| \\ &\leq \lambda(b+1)\|gu - gv\| \\ &= \|gu - gv\|. \end{aligned}$$

Thus, $f_{g,\lambda}$ is g -nonexpansive. Let $\{k_n\} \subset (0, 1)$ satisfy $k_n \rightarrow 1$, and define

$$f_n u = (1 - k_n)q + k_n f_{g,\lambda} u, \quad u \in \mathcal{M}.$$

Since f and g map $\mathcal{M}$ into itself and $\mathcal{M}$ is convex, $f_{g,\lambda}(\mathcal{M}) \subseteq \mathcal{M}$. Moreover, $f(\mathcal{M}) \subseteq g(\mathcal{M})$, $g(\mathcal{M})$ is convex, and $q = gq \in g(\mathcal{M})$. Hence, $f_{g,\lambda}(\mathcal{M}) \subseteq g(\mathcal{M})$,

*Corresponding author. Manahell Alsosui

2020 Mathematics Subject Classification. 47H10, 54H25.

Keywords and Phrases. ultraoccasionally weakly compatible map, enriched g -nonexpansive mapping, hemicompact mapping, common fixed point, point of coincidence

and therefore,

$$f_n(\mathcal{M}) \subseteq \mathcal{M} \cap g(\mathcal{M}).$$

Moreover,

$$\|f_n u - f_n v\| \leq k_n \|gu - gv\|, \quad u, v \in \mathcal{M}.$$

Thus, each f_n is a $(0, k_n)$ -enriched g -contraction, equivalently, a g -contraction.

Set

$$c_n = 1 - k_n + k_n \lambda, \quad r_n = \frac{k_n \lambda}{c_n}.$$

Then $0 < r_n < 1$ and

$$f_n = c_n [(1 - r_n)q + r_n f] + k_n (1 - \lambda)g.$$

For clarity, define

$$h_{r_n} = (1 - r_n)q + r_n f.$$

By the ultraoccasionally weak compatibility assumption of f and g , in the sense of Definition 8 of the original article, h_{r_n} and g are occasionally weakly compatible.

Thus, there exists $v_n \in \mathcal{M}$ such that

$$h_{r_n} v_n = g v_n \quad \text{and} \quad h_{r_n} g v_n = g h_{r_n} v_n.$$

Since

$$c_n + k_n (1 - \lambda) = 1,$$

we obtain

$$f_n v_n = c_n h_{r_n} v_n + k_n (1 - \lambda) g v_n = g v_n.$$

Also,

$$\begin{aligned} f_n g v_n &= c_n h_{r_n} g v_n + k_n (1 - \lambda) g^2 v_n \\ &= c_n g h_{r_n} v_n + k_n (1 - \lambda) g^2 v_n \\ &= g^2 v_n = g f_n v_n. \end{aligned}$$

Therefore, f_n and g are occasionally weakly compatible for every n . By Theorem 6, there exists $u_n \in \mathcal{M}$ such that

$$u_n = f_n u_n = g u_n.$$

Consequently,

$$[1 - k_n (1 - \lambda)] u_n = (1 - k_n) q + k_n \lambda f u_n.$$

Let $B > 0$ satisfy $\|f u\| \leq B$ for all $u \in \mathcal{M}$. Since $1 - k_n (1 - \lambda) \geq \lambda > 0$, the preceding identity yields

$$\|u_n\| \leq \frac{(1 - k_n) \|q\| + k_n \lambda B}{1 - k_n (1 - \lambda)} \leq \frac{\|q\| + \lambda B}{\lambda}.$$

Thus, $\{u_n\}$ is bounded. Since $gu_n = u_n$,

$$f_{g,\lambda}u_n = (1 - \lambda)u_n + \lambda fu_n,$$

so $\{f_{g,\lambda}u_n\}$ is also bounded. Therefore,

$$u_n - f_{g,\lambda}u_n = (1 - k_n)(q - f_{g,\lambda}u_n) \longrightarrow 0.$$

Since $gu_n = u_n$,

$$u_n - f_{g,\lambda}u_n = \lambda(u_n - fu_n).$$

Thus, $\|u_n - fu_n\| \rightarrow 0$. Since $\{u_n\}$ is bounded and f is hemicompact, there exist a subsequence $\{u_{n_j}\}$ and a point $a \in \mathcal{X}$ such that $u_{n_j} \rightarrow a$. For every j , $u_{n_j} = gu_{n_j} \in g(\mathcal{M})$. Since $g(\mathcal{M})$ is complete, it is closed in $\mathcal{X}$. Hence, $a \in g(\mathcal{M}) \subseteq \mathcal{M}$. The continuity of f , together with $u_{n_j} - fu_{n_j} \rightarrow 0$, gives $fa = a$. Also, $gu_{n_j} = u_{n_j} \rightarrow a$, and the continuity of g gives $ga = a$. Hence,

$$fa = a = ga,$$

and therefore, $Fix(f, g) \neq \emptyset$. Thus, Theorem 7 remains valid with the corrected auxiliary construction above.

The following additional corrections should also be made.

1. In the statements of Theorems 5 and 6 and Corollary 2, replace “a unique coincidence point” by “a unique point of coincidence.” In the proof of Theorem 5, replace the condition $z \neq w$ by $gz \neq gw$. The sentence

“Thus, $gz = gw = a$ which shows that $C(f, g)$ is nonempty and a is unique.”

should read

“Thus, $gz = gw = a$, and hence, a is the unique point of coincidence of f and g .”

2. In Theorem 8, delete the assumption that $f(B_{\mathcal{M}}(p))$ is bounded. Indeed, the proof gives $f(B_{\mathcal{M}}(p)) \subseteq B_{\mathcal{M}}(p)$, and $B_{\mathcal{M}}(p)$ is bounded. Furthermore, replace $q \in Fix(g)$ by $q \in Fix(g) \cap B_{\mathcal{M}}(p)$, and require ultraoccasionally weak compatibility of f and g on $B_{\mathcal{M}}(p)$ with respect to this q .

At the beginning of the proof of Theorem 8, add the following argument. If $p \in \mathcal{M}$, then $p \in CAP(f, g)$ gives $fp = gp = p$, and hence, $p \in Fix(f, g) \cap B_{\mathcal{M}}(p)$. Thus, one may assume $p \notin \mathcal{M}$. Since $q \in B_{\mathcal{M}}(p)$, one then has $\text{dist}(p, \mathcal{M}) > 0$, so the strict inequality used to prove $B_{\mathcal{M}}(p) \subseteq \partial\mathcal{M} \cap \mathcal{M}$ is valid. Theorem 7 therefore applies on $B_{\mathcal{M}}(p)$.

3. In Subsection 4.2, let $I \subseteq \mathbb{R}$ be a nonempty bounded interval and put

$$\mathcal{C}_I = \{\psi \in C([0, a], \mathbb{R}) : \psi([0, a]) \subseteq I\}.$$

The operators in Theorem 9 are to be considered on $\mathcal{C}_I$. Assume that A_1, A_2 are bounded members of

$$C([0, a] \times [0, a] \times I, \mathbb{R})$$

and that the operators defined by the original integral formulas satisfy

$$f, g : \mathcal{C}_I \rightarrow \mathcal{C}_I.$$

In condition (i), replace $\psi_i(t)$, $(g\psi_i)(t)$, and $(f\psi)(t)$ by $\psi_i(s)$, $(g\psi_i)(s)$, and $(f\psi)(s)$, respectively, for $\psi_1, \psi_2, \psi \in \mathcal{C}_I$. Condition (ii) of Theorem 9 should read as follows:

$$f(\mathcal{C}_I) \subseteq g(\mathcal{C}_I), \quad g(\mathcal{C}_I) \text{ is complete in the Bielecki norm.}$$

The convergence assertion for iteration (11) is to be stated for every initial function $\psi_0 \in \mathcal{C}_I$; equivalently, choose $\psi_{n+1} \in \mathcal{C}_I$ so that $g\psi_{n+1} = f\psi_n$.

4. In the proof of Theorem 9, use $f(\mathcal{C}_I) \subseteq g(\mathcal{C}_I)$. The set $\mathcal{C}_I$ is convex, and the affinity of A_2 in its third variable makes g affine; hence, $g(\mathcal{C}_I)$ is convex. Theorem 6, with $b = 0$, first gives a unique point of coincidence. If $\hat{\psi}$ is a coincidence point, then

$$\|fg\hat{\psi} - gf\hat{\psi}\|_\beta \leq \|g\hat{\psi} - f\hat{\psi}\|_\beta = 0.$$

Thus, f and g are occasionally weakly compatible, and the second part of Theorem 6 yields the unique common solution and convergence of iteration (11) for every $\psi_0 \in \mathcal{C}_I$.

Manahell Alsosui

Department of Mathematics,
Taibah University,
Madinah, Saudi Arabia

E-mail: msousy@yahoo.com

<https://orcid.org/0009-0001-9103-9159>

Naseer Shahzad

Department of Mathematics,
King Abdulaziz University,
P.O. Box 80203,
Jeddah 21589, Saudi Arabia

E-mail: nshahzad@kau.edu.sa

<https://orcid.org/0000-0001-7155-5917>

Amer Hassan Albargi

Department of Mathematics,
King Abdulaziz University,
P.O. Box 80203,
Jeddah 21589, Saudi Arabia

E-mail: aalbarqi@kau.edu.sa

<https://orcid.org/0009-0003-6578-7908>

(Received 17.10.2026.)

(Revised 18.10.2026.)